A SAT Solver and Computer Algebra Attack on the Minimum Kochen–Specker Problem (Student Abstract)

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Full Paper

The Kochen–Specker Theorem

- Finding the minimum size of a **Kochen–Specker vector system** has been an open problem in quantum foundations for over 50 years.
- We present the first implementation of a SAT+CAS tool (a combination of a SAT solver and a Computer Algebra System, along with extensive proof verification) to tackle this problem.
- We obtain a tighter lower bound with **four orders of magnitude speedup** over previous methods.

Authors	Year	Bound
Kochen, Specker	1967	≤ 117
Jost	1976	≤ 109
Conway, Kochen	1990	≤ 31
Arends, Ouaknine, Wampler	2009	≥ 18
Uijlen, Westerbaan	2016	≥ 22
Li, Bright, Ganesh	2022	≥ 23
Li, Bright, Ganesh / Kirchweger, Peitl, Szeider	2023	≥ 24

Table 1. A chronology of the bounds on the size of the minimum KS vector system in three dimensions.

SAT Encoding of the Kochen–Specker Graphs

- A graph is **embeddable** if every pair of adjacency vertices can be mapped to orthogonal vectors.
- A graph is **010-colorable** if there is a $\{0,1\}$-coloring of the vertices such that no two adjacent vertices are colored 1 and the vertices are not all colored 0 in each triangle.
- A **KS graph** is an embeddable and non-010-colorable graph, and the minimum KS problem is to find the smallest KS graph.
- We encode graph properties of KS graph into a SAT instance in conjunctive normal form (CNF).

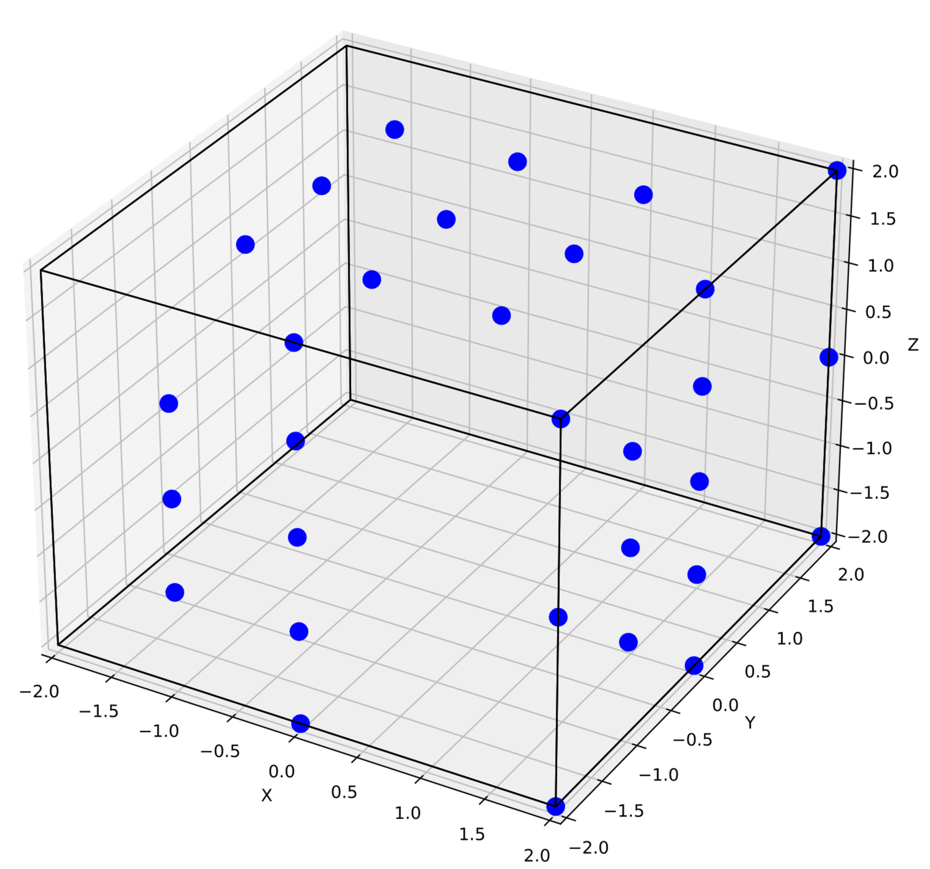

Figure 1. The 31 vectors of the smallest known KS system in three dimensions (discovered by John Conway and Simon Kochen circa 1990).

Orderly Generation via SAT+CAS

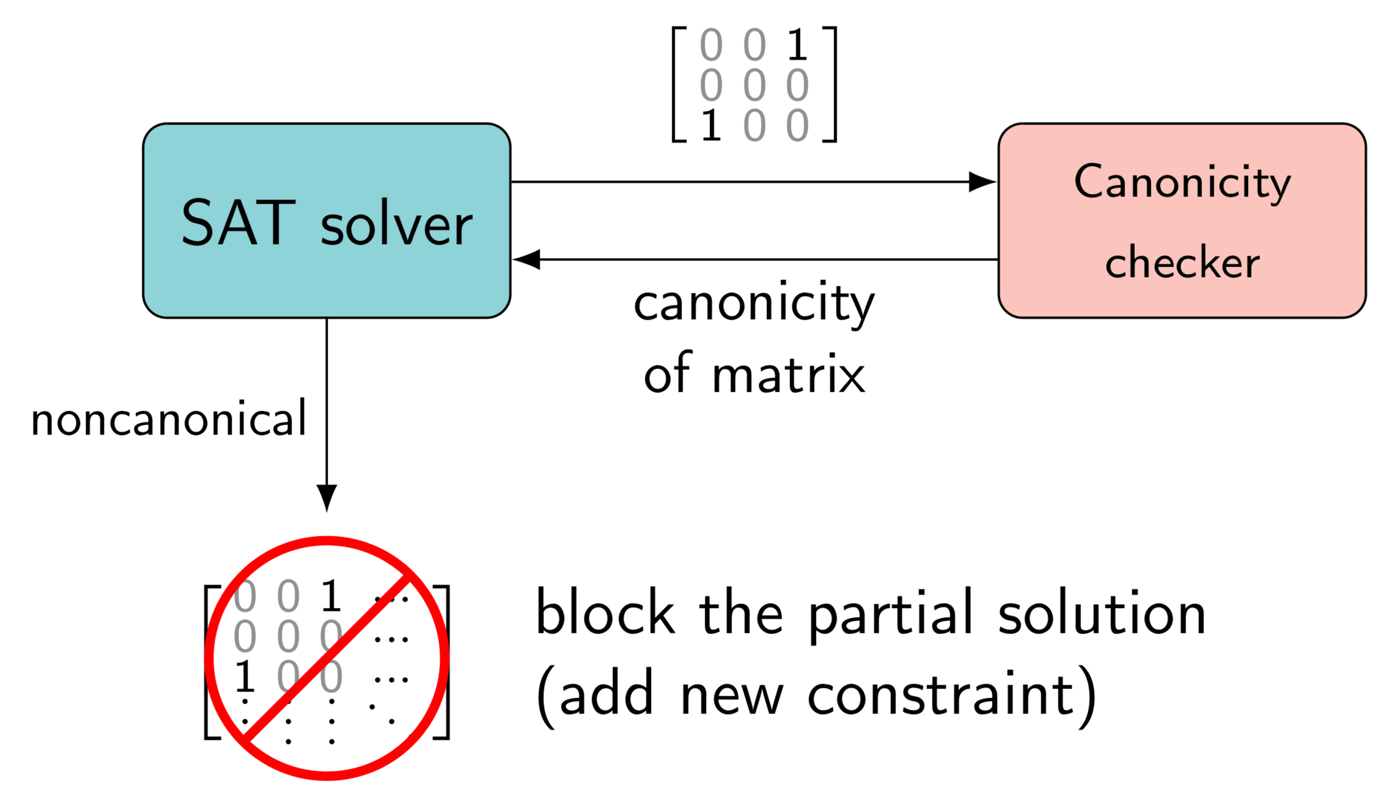

Figure 2. A flowchart of orderly generation algorithm in PhysicsCheck's SAT+CAS architecture.

- Each set of isomorphic graphs has a unique canonical representative.
- The SAT + CAS technique learns "blocking" clauses that prevent searching for noncanonical graphs.

Sequential PhysicsCheck

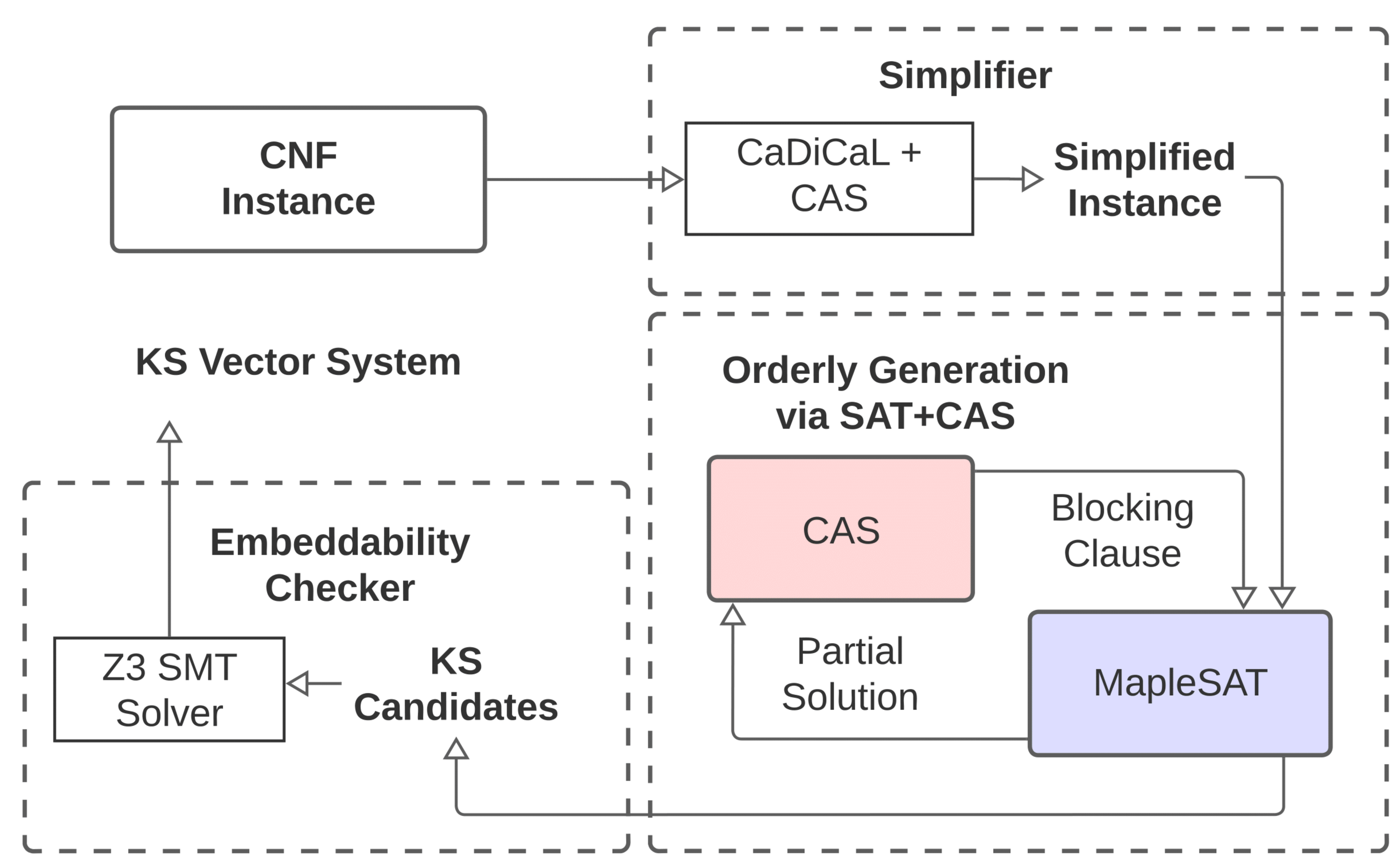

Figure 3. A flowchart of our SAT+CAS based tool PhysicsCheck for solving the KS problem in the sequential setting.

Parallel PhysicsCheck

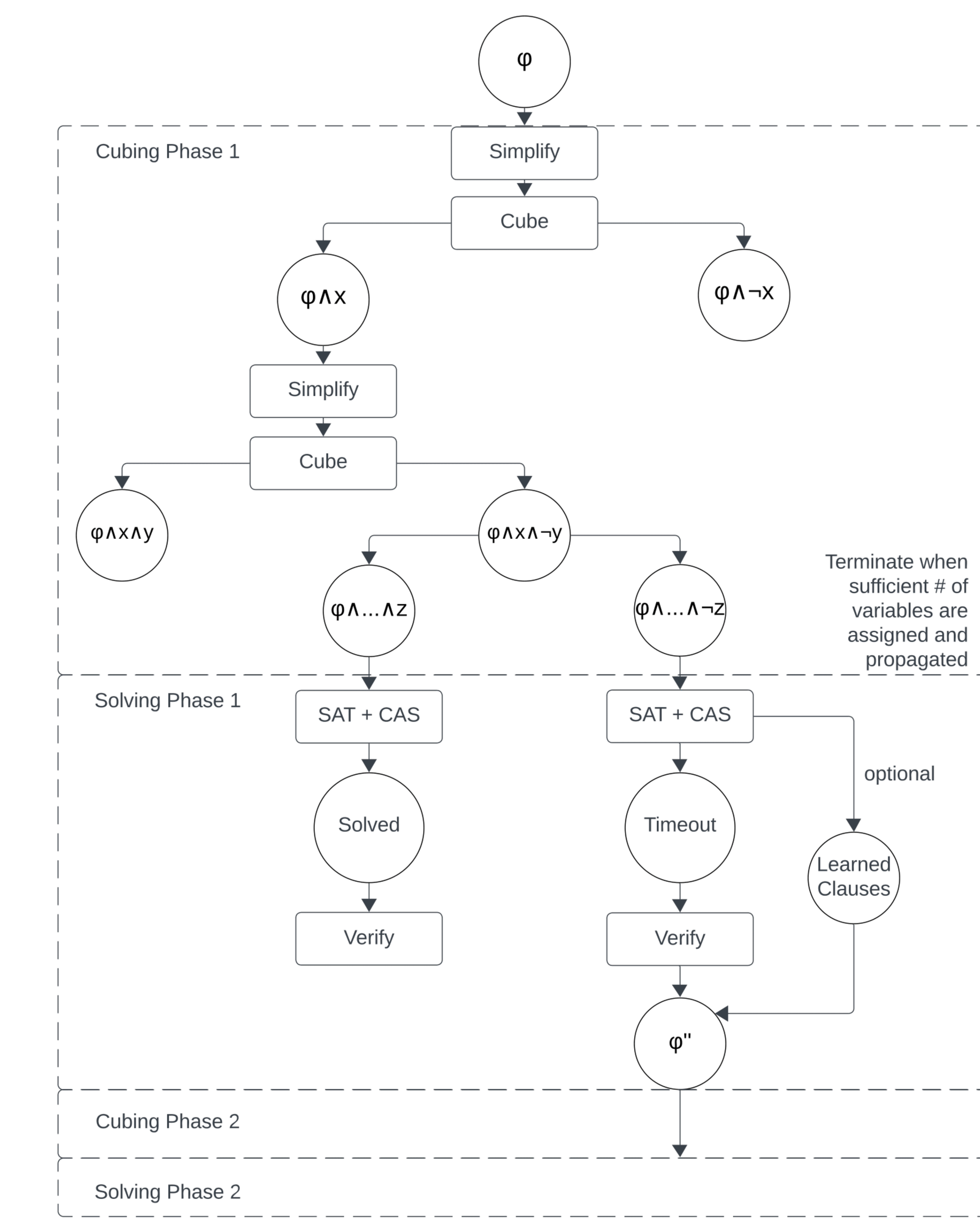

Figure 4. A flowchart of our SAT+CAS based tool PhysicsCheck for solving the KS problem in the parallel setting.

- During the cubing phase, the cubing solver **march_cu** is used to find the next variable to branch on, while **CaDiCaL** is used to simplify the instance at each iteration.
- During the solving phase, each subproblem is solved using **MapleSAT + CAS** and verified using **DRAT-trim**.

Results and Verification

order	Candidates	SAT+CAS	Speedup over SAT	Speedup over CAS
17	1	0.02 h	8.4×	24.2×
18	0	0.04 h	123.8×	211.5×
19	8	0.22 h	883.5×	717.6×
20	147	1.35 h	timeout	timeout
21	2,497	18.12 h	timeout	timeout
22	88,282	356.88 h	timeout	timeout
23	3,747,950	52,619.16 h	timeout	timeout

Table 2. Speedup of SAT+CAS over SAT-only and CAS-only. Order 23 was solved with a cube-and-conquer approach.

- All KS candidates of order less than 24 are not embeddable. Therefore, we conclude that the minimum size of a KS system is **at least 24**.
- The uncompressed proofs in order 22 are about **1.9 TiB** in total, and **41.6 TiB** for order 23.

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