# CanaDAM 2025
# Computer Assisted Mathematics Minisymposium

**Computational bounds for book Ramsey numbers**
William J. Wesley, University of California, San Diego

Given graphs $G_1, \ldots, G_r$, the *Ramsey number* $R(G_1, \ldots, G_r)$ is the smallest $n$ such that if the edges of $K_n$ are $r$-colored, there is a copy of $G_i$ in color $i$ for some $i$. The most famous cases are the "classical" Ramsey numbers $R(K_s, K_t)$, which are notoriously difficult to compute exactly, but other classes of graphs such as books, cycles, and wheels have also received significant interest.

This talk will focus on new lower and upper bounds obtained for the Ramsey numbers $R(B_m, B_n)$, where $B_n = K_2 + \overline{K_n}$ is a *book graph*. Most of these results are computational and obtained by Boolean satisfiability (SAT) or integer programming (IP) solvers, but we also give an explicit construction that yields an infinite family of tight lower bounds. We also give new bounds for some three color Ramsey numbers that arise from SAT solving and constructions using nonabelian groups.

**A Computational Counterexample on Sets Containing Fibonacci Numbers**
Karyn McLellan, Mount Saint Vincent University

At the Sixteenth International Conference on Fibonacci Numbers and Their Applications, the following problem was posed: Let S be the set generated by these rules: Let $1 \in S$ and if $x \in S$, then $2x \in S$ and $1 - x \in S$, so that $S$ grows in generations: $G_1 = \{1\}$, $G_2 = \{0, 2\}$, $G_3 = \{-1, 4\}$, $\ldots$. Prove or disprove that each generation contains at least one Fibonacci number or its negative. We will show that every integer $k$ can be found in some $G_i$ and will disprove the above statement by finding an expression for the generation index $i$ for any given $k$. We will use a variety of recurrence sequences including the dragon curve sequence, properties of binary numbers, and a computer calculation to find numerous counterexamples. (Joint work with Danielle Cox, MSVU)

**Formalizing Combinatorics Definitions in the Lean Theorem Prover**
Alena Gusakov, University of Waterloo

Formalizing mathematical theories in a proof assistant is a challenging task, of which the most difficult part is, counterintuitively, formalizing definitions. There are many considerations one has to make, such as level of generality, readability, and ease of use in the type system, and there are typically multiple equivalent or related definitions from which to choose. Inclusion of a formal mathematical definition in a centralized community-run mathematical library is typically an indication that the definition is "good."

We will explore the motivations for formalizing mathematics, make some observations about what makes a formalized definition "good," and examine case studies of the process of writing and editing definitions that have ultimately been added to the Lean Theorem Prover community-run mathematical library, mathlib.

**Formalizing a result on polygonal numbers in Lean 4—an experience report**

Kevin Cheung, Carleton University

Since the computer-assisted proof of the Four-Colour Theorem by Appel and Haken, the role of computers in mathematical proofs now spans from performing exhaustive search and case analyes to providing ways for proof-discovery as well as formal verification. In this talk, I will describe a few surprises that arose in a joint effort with Tomas NcNamer to formalize in Lean 4 Melvyn B. Nathanson's short proof of Cauchy's Polygonal Number Theorem.

**The maximum number of mutually orthogoval Desarguesian affine planes of order $2^n$**

Jonathan Jedwab, Simon Fraser University

Two affine planes of the same order and on the same pointset are *orthogoval* if each line of one plane intersects each line of the other plane in at most two points. A set of pairwise orthogoval affine planes is *mutually orthogoval*.

I shall describe a new computational method to determine the maximum number of Desarguesian mutually orthogoval affine planes of order $2^n$. This gives exact results for orders 8 and 16, and indicative results for orders 32 and 64.

This is joint work with James A. Davis, Shuxing Li, Jingzhou Na, Thomas Pender, and Tabriz Popatia.